

Category Theory (Solution)

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The morphisms

The word transformations are as follows:

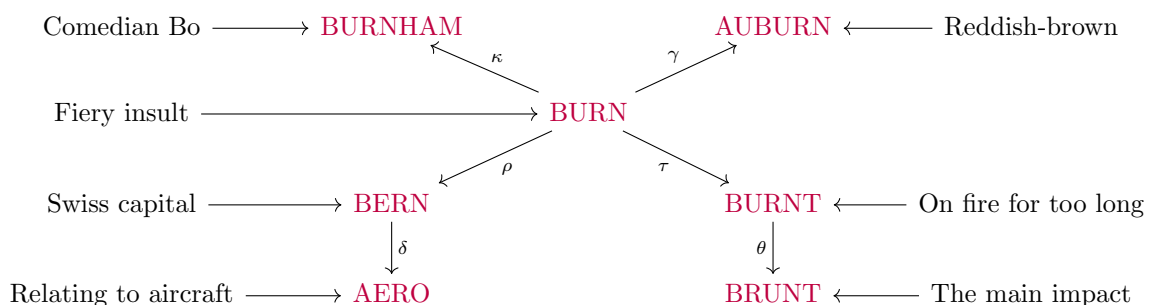
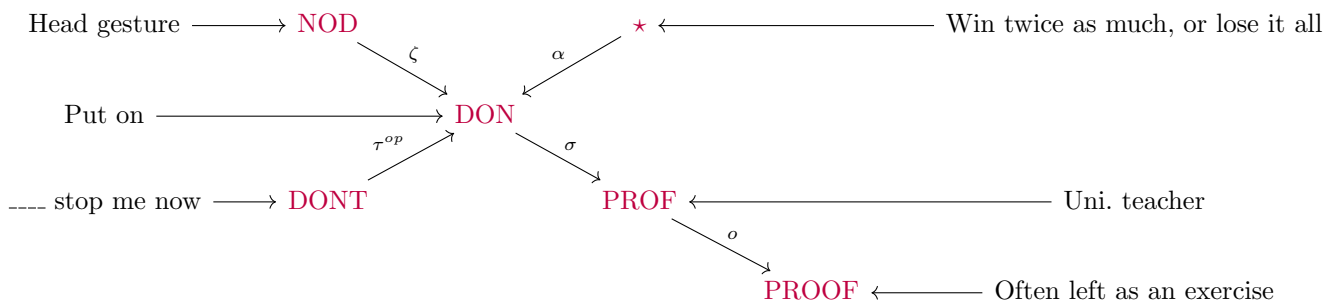
- α : Acronym (take initial letters)
- β : Example of
- γ : Add AU to the start
- δ : Caesar shift one letter forward by 1, another letter backward by 1
- ζ : Reverse
- η : Add a group of three letters anywhere
- θ : Switch two letters
- ι : Add a word to the start
- κ : Add a word to the end
- \omicron : Duplicate O
- ρ : Homophone
- σ : Synonym
- τ : Add T to the end

If a morphism has op , then the operation should be done in reverse. For example, τ^{op} would remove a T from the end.

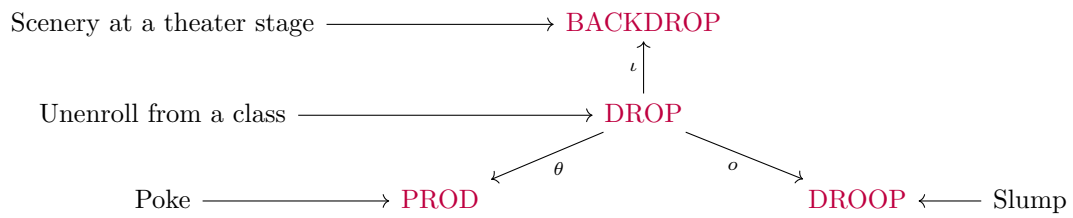
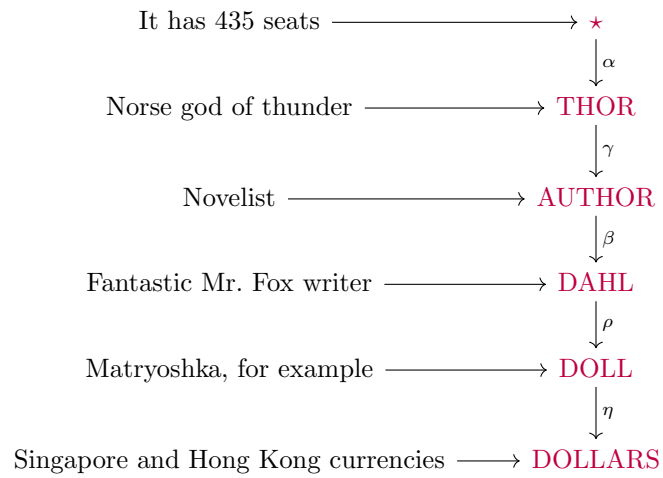
Practice

Solving these isn't necessary, but could help you learn the morphisms.

The \star is "DOUBLE OR NOTHING". It was too long to fit in the diagram.



The ★ is “THE HOUSE OF REPRESENTATIVES”.



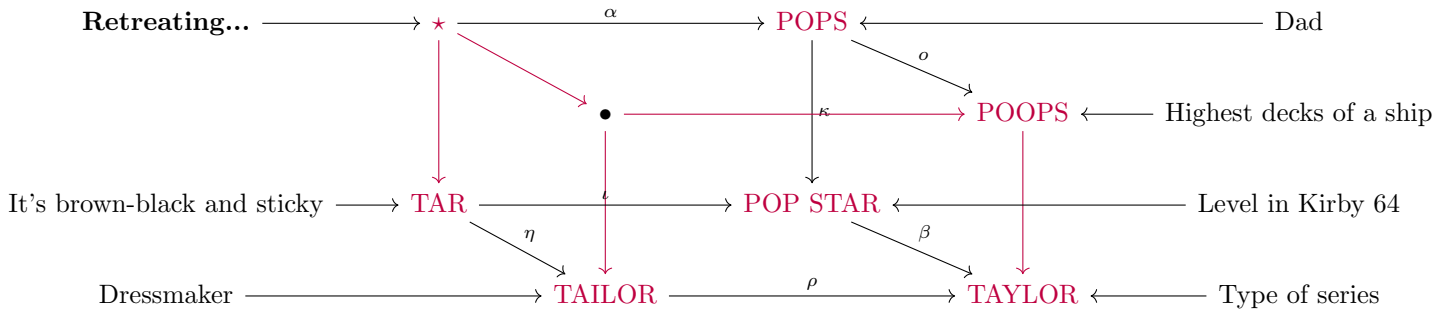
Homework

Instead of looking up diagrams of the jail you're escaping, you should boldly look up category theory diagrams and compare them to these diagrams.

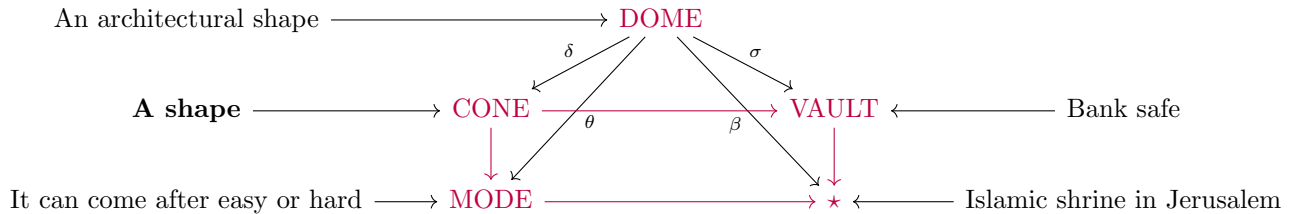
In each diagram, the bold clue's answer is a category theory term, which is associated with a diagram (e.g. the diagram represents the word, or the diagram shows an axiom relating to the word). The diagrams presented here are incomplete; they are missing arrows from the real category theory diagrams.

The key step is filling in the missing arrows for each diagram. The shapes of the missing arrows (drawn in purple) spell out "MONOID".

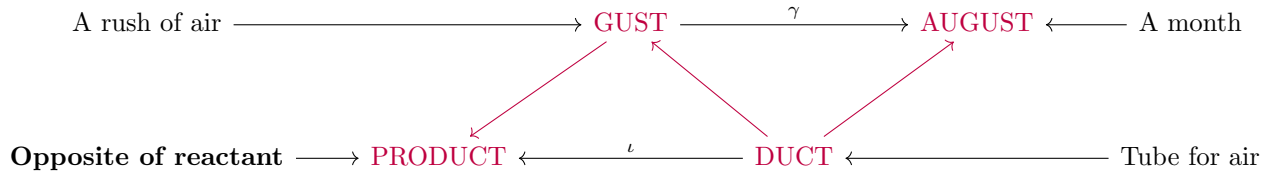
The \star is "PULLBACK OF PULLBACK SQUARE". Click here for a picture of it. The "Retreating..." clue was "Retreating from a plaza of retreating people", shortened here due to lack of space.



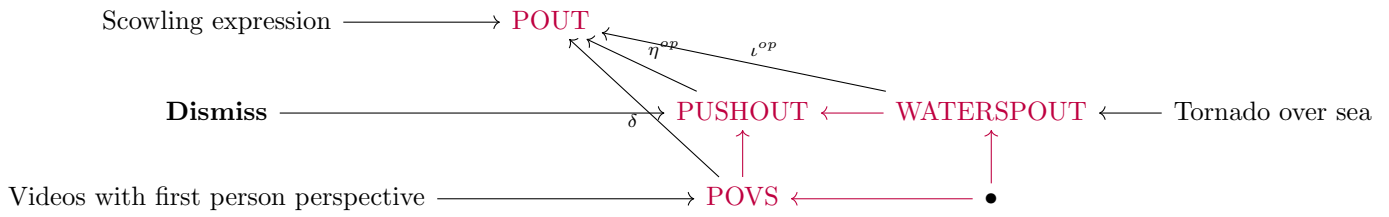
The \star is "DOME OF THE ROCK". Click here for a picture of a cone in category theory.



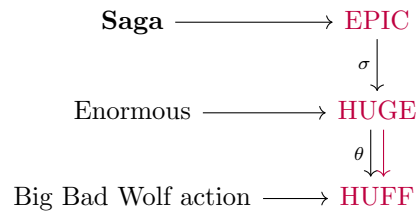
Click here for a picture of a product in category theory. This is a binary product.



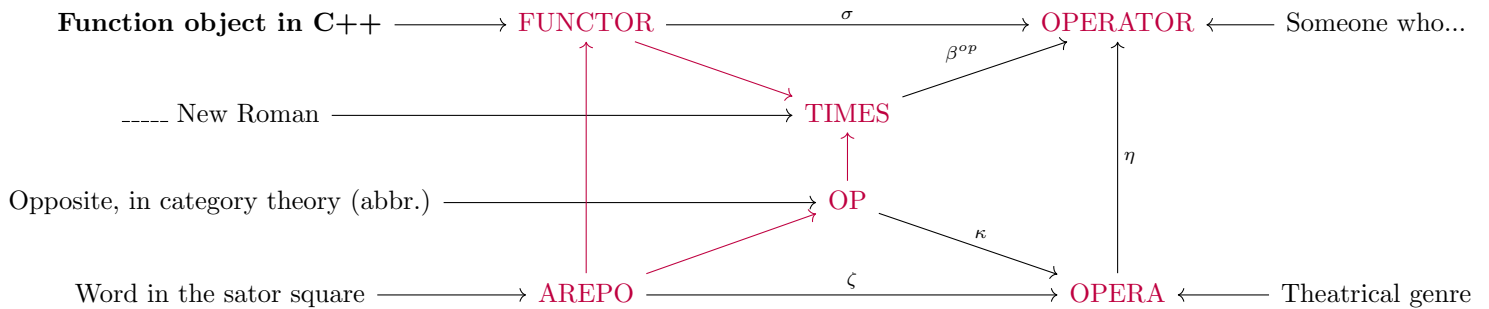
Click here for a picture of a pushout.



Click here for a picture of an epic, or epimorphism.



Click here for a picture of a functor. The diagram showcases the composition axiom for functors. The “Someone who...” clue was “Someone who helps direct phone calls”, shortened here due to lack of space.



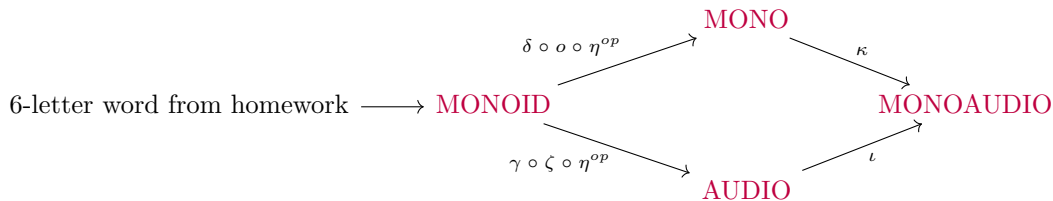
Exam

MONOID was the 6-letter word from homework. When we compose two functions together, we do the right function, then the left function. Also, the intermediate steps might not be actual English words.

For the top, we first do the transformation η^{op} , then o , then δ . Thus $\text{MONOID} \rightarrow \text{MON} \rightarrow \text{MOON} \rightarrow \text{MONO}$.

For the bottom, we first do η^{op} , then ζ , then γ . Thus $\text{MONOID} \rightarrow \text{OID} \rightarrow \text{DIO} \rightarrow \text{AUDIO}$.

Finally, we put together the two words to get MONOAUDIO, the final answer!



Author's Notes

I'd like to thank my category theory professor for drawing all sorts of funny pictures that inspired this puzzle. I'd also like to thank math3ma for having a fantastic blog about all sorts of mathematics, including category theory. Finally, I used Yichuan Shen's tikzcd editor to make all the diagrams in this puzzle.

A team informed us that the diagram for "CONE" technically already is a cone (over a diagram with 4 objects and no arrows between them)! Oops. Perhaps I should study harder.

It's been a while since puzzlehunt CMU has used \LaTeX to typeset puzzles. I wanted to evoke the wide variety of emotions upon seeing abstract nonsensical math in a puzzle. Some other interesting words in category theory include:

- Fiber product
- Adhesive
- Full and faithful functors
- Equalizer
- Initial and terminal objects
- Free category
- Cocone
- Meet and join
- Natural transformations (not word transformations)